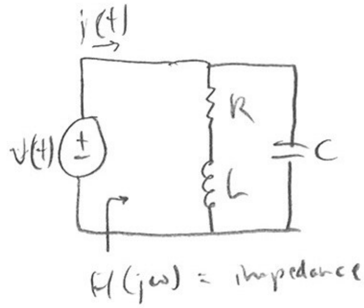
This is a homework problem from when I took signals

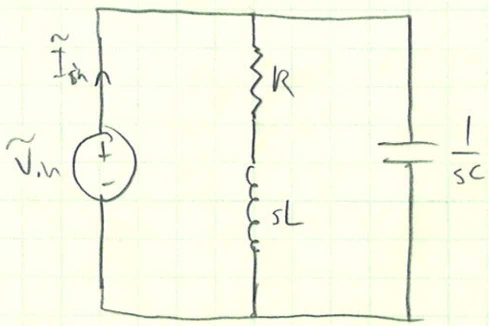


v and i are sinusoidal.

There is a frequency ω_r at which v is in phase with i.
There is another possibly different frequency ω_0 at which i is minimized (the impedance is maximized.

Determine whether $\omega_r = \omega_0$, $\omega_r > \omega_0$, $\omega_r < \omega_0$, or none of them (e.g. sometimes $\omega_r > \omega_0$ and sometimes $\omega_r < \omega_0$).

2.



$$H(s) = \frac{\tilde{V}_{in}}{\tilde{I}_{in}} = (R + sL) \,//\, \frac{1}{sC} = \frac{R + sL}{sC\left(R + sL + \frac{1}{sC}\right)} = \frac{sL + R}{s^2LC + sRC + 1}$$

If the phase of $H(j\omega)$ is zero, then $H(j\omega)$ is real.

Equivalently,

$$H(j\omega) = \overline{H(j\omega)}$$

$$\frac{j\omega L + R}{-\omega^2 LC + j\omega RC + 1} = \frac{-j\omega L + R}{-\omega^2 LC - j\omega RC + 1}$$

$$-j\omega^3L^2C + \omega^2 \cancel{RLC} + j\omega L - \omega^2 \cancel{RLC} - j\omega R^2C + \cancel{R} =$$
$$j\omega^3L^2C + \omega^2\cancel{RLC} - j\omega L - \omega^2\cancel{RLC} + j\omega R^2C + \cancel{R}$$

$$2j\omega^3L^2C - 2j\omega L + 2j\omega R^2C = 0$$

$$\omega^2L^2C - (L - R^2C) = 0$$

$$\omega^2 = \frac{1}{L^2C}\left(-R^2C + L\right) = \omega_r^2$$

$H(j\omega_r)$ is real (phase $= 0$).

$$|H(j\omega)|^2 = H(j\omega)\,\overline{H(j\omega)} = \frac{j\omega L + R}{-\omega^2LC + j\omega RC + 1} \cdot \frac{-j\omega L + R}{-\omega^2LC - j\omega RC + 1}$$

$$= \frac{\omega^2L^2 + R^2}{(1 - \omega^2LC)^2 + \omega^2(RC)^2}$$

$\boxed{\text{2. cont.}}$

$$|H(j\omega)|^2 = \frac{\omega^2 L^2 + R^2}{(1-\omega^2 LC)^2 + \omega^2 (RC)^2}$$

$|H(j\omega)|^2$ will be a maximum at the same ω as $|H(j\omega)|$, because x^2 is monotonic increasing for $x \geq 0$ and $|H(j\omega)| \geq 0$.

$$\frac{d}{d\omega}\left(|H(j\omega)|^2\right) = 0 =$$

$$\frac{2\omega L^2 \left(\left((1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right) - (\omega^2 L^2 + R^2)\left(2(1-\omega^2 LC)(-2\omega LC) + 2\omega R^2 C^2\right)\right)}{\left((1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right)^2}$$

$$\cancel{2\omega} L^2 \left((1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right) = \cancel{2\omega}(\omega^2 L^2 + R^2)\left(R^2 C^2 - 2LC(1-\omega^2 LC)\right)$$

$$\cancel{L^4 C^2 \omega^4} + \cancel{L^2 C(R^2 C - 2L)\omega^2} + L^2 =$$
$$\cancel{2} L^4 C^2 \omega^4 + \cancel{L^2 C(R^2 C - 2L)\omega^2} + 2(RLC)^2 \omega^2 + R^2 C^2 (R^2 C - 2L)$$

$$L^4 C^2 \omega^4 + 2(RLC)^2 \omega^2 + R^2 C(R^2 C - 2L) - L^2 = 0$$

$$\omega^2 = \frac{-2(RLC)^2 \pm \sqrt{4R^4 L^4 C^4 - 4L^4 C^2\left(R^2 C(R^2 C - 2L) - L^2\right)}}{2L^4 C^2}$$

$$\omega_0^2 = \frac{1}{L^2 C}\left(-R^2 C + \sqrt{L^2 + 2R^2 LC}\right)$$

the negative square root is thrown out because $\omega_0^2 \geq 0$.

$|H(j\omega_0)|$ is maximized.

The ω for $H(j\omega)$ real is:

$$\omega_r^2 = \frac{1}{L^2 C}\left(-R^2 C + \sqrt{L^2 + 0}\right)$$

Since $2R^2 LC > 0$, $\omega_r < \omega_0$. The frequency for purely resistive impedance is always less than the frequency for maximum impedance.